Kent and Ingalls have written several papers about multilayer capacitors that are not documented well enough to be reproduced. Their papers have used single transmission-line models obtained by folding single transmission lines to conform to the geometry of multilayer capacitors. It seems unlikely that a single transmission-line model can be correct when a type-B connection is used because single transmission lines have but one velocity of propagation. The impedance of the multilayer transmission lines and multilayer capacitors depends upon all of the velocities of propagation as well as the exact geometry of the plates and dielectrics. The matrices used in the above paper[1] were the results of field calculations based upon the geometry and the physical properties of the materials used as dielectrics.

Recently, I have made a complete 3-D full-wave analysis of a two-plate multilayer capacitor with a type-B connection, including radiation effects. The results of that investigation show the models presented in the above paper[1] to be accurate in the frequency range used. If transmission line models of multilayer capacitors are as valuable as Kent and Ingalls' comments indicate, it is important to use models of multilayer capacitors based upon Maxwell's equations, geometry, and the physical properties of the dielectrics as a function of frequency.

References

[1] L. A. Pipes, "Matrix theory of multiconductor transmission lines," *Phil. Mag.* vol. 24, sec. 7, no. 159, pp. 97–113, July 1937.

Comments on "A New Reciprocity Theorem"

Akhlesh Lakhtakia

In the above paper,[1] a "new" reciprocity theorem for free space (i.e., vacuum) has been reported. However, it is not new, having been published in 1992 by [1]. This may be ascertained by comparing (24) of the above paper[1] with [1, Eq. (8)]. The "new" reciprocity theorem was extended in 1989 for chiral media (see [2] and [3]).

References

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[2] A. Lakhtakia, V. K. Varadan, and V. V. Varadan, *Time-Harmonic Electromagnetic Fields in Chiral Media*, Heidelberg, Germany: Springer-Verlag, 1989, pp. 22–24.

[3] A. Lakhtakia, *Beltrami Fields in Chiral Media.* Singapore: World Scientific, 1994, pp. 140–146.

Manuscript received April 6, 1996.

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Publisher Item Identifier S 0018-9480(97)00887-9.

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Comments on "A New Reciprocity Theorem"

Hristos T. Anastassiu and John L. Volakis

We write this to point out that the result in (28) of the above paper[1] can be derived very easily using a standard identity, thus eliminating the lengthy analysis originally presented. We also note that the factor of 1/2 in (26) is in error and must be deleted.

We begin the proof of (28) in the above paper starting with the identity (Gauss' theorem)

$$\int_V \nabla \cdot (\mathbf{E}_1 \times \mathbf{E}_2 - \eta^2 \mathbf{H}_1 \times \mathbf{H}_2)\, d^3v = \oint_S (\mathbf{E}_1 \times \mathbf{E}_2 - \eta^2 \mathbf{H}_1 \times \mathbf{H}_2) \cdot \hat{\mathbf{n}}\, d^2S. \quad (1)$$

As in the above paper, $(\mathbf{E}_1, \mathbf{H}_1)$ and $(\mathbf{E}_2, \mathbf{H}_2)$ are the fields associated with the corresponding sources $(\mathbf{J}_1, \mathbf{M}_1)$ and $(\mathbf{J}_2, \mathbf{M}_2)$. Also, η is the intrinsic impedance of the medium.

Next, we invoke the standard vector identity

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot \nabla \times \mathbf{A} - \mathbf{A} \cdot \nabla \times \mathbf{B} \quad (2)$$

to obtain

$$\nabla \cdot (\mathbf{E}_1 \times \mathbf{E}_2 - \eta^2 \mathbf{H}_1 \times \mathbf{H}_2) = \mathbf{E}_2 \cdot \nabla \times \mathbf{E}_1 - \mathbf{E}_1 \cdot \nabla \times \mathbf{E}_2 - \eta^2 \mathbf{H}_2 \cdot \nabla \times \mathbf{H}_1 + \eta^2 \mathbf{H}_1 \cdot \nabla \times \mathbf{H}_2 \quad (3)$$

which upon using Maxwell's equations can be rewritten as

$$\nabla \cdot (\mathbf{E}_1 \times \mathbf{E}_2 - \eta^2 \mathbf{H}_1 \times \mathbf{H}_2) = \mathbf{E}_1 \cdot \mathbf{M}_2 - \mathbf{E}_2 \cdot \mathbf{M}_1 + \eta^2 \mathbf{H}_1 \cdot \mathbf{J}_2 - \eta^2 \mathbf{H}_2 \cdot \mathbf{J}_1. \quad (4)$$

Substituting the latter expression into (1) yields

$$\int_V (\mathbf{E}_1 \cdot \mathbf{M}_2 - \mathbf{E}_2 \cdot \mathbf{M}_1 + \eta^2 \mathbf{H}_1 \cdot \mathbf{J}_2 - \eta^2 \mathbf{H}_2 \cdot \mathbf{J}_1)\, d^3v = \oint_S (\mathbf{E}_1 \times \mathbf{E}_2 - \eta^2 \mathbf{H}_1 \times \mathbf{H}_2) \cdot \hat{\mathbf{n}}\, d^2S. \quad (5)$$

Finally, as S goes to infinity the right-hand side (RHS) of (5) vanishes and thus the authors obtain

$$\int_{V_\infty} (\mathbf{E}_1 \cdot \mathbf{M}_2 + \eta^2 \mathbf{H}_1 \cdot \mathbf{J}_2)\, d^3v = \int_{V_\infty} (\mathbf{E}_2 \cdot \mathbf{M}_1 + \eta^2 \mathbf{H}_2 \cdot \mathbf{J}_1)\, d^3v \quad (6)$$

which is identical to (28) in the above paper.[1]

Manuscript received April 6, 1996.

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Publisher Item Identifier S 0018-9480(97)00847-8.

Author's Reply

J. C. Monzon

I wish to thank Volakis and Anastassiu for their interest shown in the above comments. Before directly responding to their comments, I would like to point out that I have recently been exposed[2] to a related work by Fel'd [1]. One of the equations, namely (24) in the above paper, was derived by Fel'd by alternative means. Neither the reviewers, myself, nor Volakis and Anastassiu knew of this reference, perhaps because the *Soviet Physics Doklady* is not very accessible to American engineers, and also because of its rather unusual title: "A quadratic lemma of electrodynamics." I believe the work of Fel'd deserves recognition in this TRANSACTIONS. However, I would like to state that my work encompasses the above paper and its generalization to more complex materials [2], and was done in 1991 (under Naval Research Laboratory (NRL) sponsorship), i.e., a year earlier than the paper by Fel'd.

Volakis and Anastassiu point out two things: 1) that (26) of the above paper has a factor of 1/2 in error; and 2) that (28) of the above paper can be derived easily by alternative means.

With regard to the factor 1/2, I believe it should be there since (26) is used to augment (24) in the sense that it is added on both sides. This is done appropriately by switching indices so as to present (24) with a statement of reciprocity in the usual operator sense, i.e.,

$$\mathrm{O}_+(1,\,2) = \mathrm{O}_+(2,\,1) \tag{C1}$$

for

$$\mathrm{O}_+(1,\,2) = \int_V \mathrm{d}\tau[\overline{\mathrm{M}}^{(2)}\cdot\overline{\mathrm{E}}^{(1)} + \eta^2\overline{J}^{(2)}\cdot\overline{\mathrm{H}}^{(1)}] + \frac{\eta^2}{2}\oint_S \mathrm{d}\overline{\mathrm{S}}\cdot\left[\overline{\mathrm{H}}^{(1)}\times\overline{\mathrm{H}}^{(2)} - \frac{1}{\eta^2}\overline{\mathrm{E}}^{(1)}\times\overline{\mathrm{E}}^{(2)}\right]. \tag{C2}$$

Similarly, (25) is added to each side of (23) appropriately resulting in the usual statement of reciprocity

$$\mathrm{O}_-(1,\,2) = \mathrm{O}_-(2,\,1) \tag{C3}$$

for

$$\mathrm{O}_-(1,\,2) = \int_V \mathrm{d}\tau[\overline{J}^{(1)}\cdot\overline{\mathrm{E}}^{(2)} - \overline{\mathrm{M}}^{(1)}\cdot\overline{\mathrm{H}}^{(2)}] + \frac{1}{2}\oint_S \mathrm{d}\overline{\mathrm{S}}\cdot[\overline{\mathrm{H}}^{(1)}\times\mathrm{E}^{(2)} + \overline{\mathrm{E}}^{(1)}\times\overline{\mathrm{H}}^{(2)}]. \tag{C4}$$

It should be noted that (25) and (26) were introduced in a casual manner because they were used in an argument just to show that (23) and (24) were independent. Equations (25) and (26) are never used on their own. It is for this reason that the infinite integrals in (23) and (24) are never reduced to integrals over volume V, such as I have done above.

With respect to the derivation of (24) in the paper, I do not think that the analysis is lengthy. The analysis leading to (24) is 1-1/2 TRANSACTIONS' pages long, where (24) is not the only significant result, but also important is (19), the statement of reciprocity of the characteristic modes. It should also be noted that the introduction of characteristic modes allows the results to be obtained in a natural fashion, wherein the new theorem appears like the natural complement of the accepted form (one being the sum of the u's, the other the difference).

The fact that (24) can be derived by alternative means is known to me; an anonymous reviewer was the first to point this out to me (see the acknowledgment in the paper). Once a final result is known, it can of course be re-derived in a variety of ways. For instance, the vector used by Volakis (in (1) of the comment) was not derived and has no justification other than to duplicate (24) of the paper. What Volakis and Anastassiu present here is essentially what the anonymous reviewer presented to me, and most importantly, follows the same steps of the paper by Fel'd.

To summarize, the factor of 1/2 is not in error, and the "shorter" method presented by Volakis and Anastassiu is already available in the Soviet literature.

Manuscript received August 12, 1996.
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Publisher Item Identifier S 0018-9480(97)00848-X.

[2] Thanks are extended to A. Lakhtakia for introducing the author to the work of Fel'd.

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Corrections to "Reconstruction of the Constitutive Parameters for an Ω Material in a Rectangular Waveguide"

Martin Norgren and Sailing He

I. THE DIRECT PROBLEM

Due to a mistake, certain parts of the analysis in the above paper[1] are incorrect. Here we present the necessary corrections. It is shown that the corrected formalism leads to improved reconstructions. We consider a homogeneous block of an Ω material, filling the region $0 \le z \le L$ in a metallic rectangular waveguide with cross section $0 \le x \le a$ and $0 \le y \le b$.

To repeat, from analysis of Maxwell's equations

$$\nabla\times\vec{\mathbf{E}} = -j\omega(\overline{\overline{\mu}}\vec{\mathbf{H}} + \overline{\overline{\zeta}}\vec{\mathbf{E}}) \qquad \nabla\times\vec{\mathbf{H}} = j\omega(\overline{\overline{\epsilon}}\vec{\mathbf{E}} + \overline{\overline{\xi}}\vec{\mathbf{H}}) \tag{1}$$

with a time and z dependence of $\exp{(j\omega t - \gamma z)}$, it can be shown [1] that TE_{m0} (and TE_{0n}) modes can exist. For the TE_{m0} modes propagating in the $+z$ direction, we have the following set of solutions

$$H_3^m = C_m\cos\left(\frac{m\pi x}{a}\right)\exp{(-\gamma_m z)} \tag{2}$$

Manuscript received June 8, 1996.
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Publisher Item Identifier S 0018-9480(97)00846-6.

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